IBEHS 3A03 Assignment #1: Properties of Systems

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[**PURPOSE**](#_ad1xb01rvaok) **3**

[**METHODOLOGY**](#_wn6tj5j1tm86) **3**

[Causality Test](#_ej4c55aagh1h) 3

[Memory Test](#_4euadb731lu7) 3

[Linearity Test](#_ablryc612c6p) 4

[Time Invariance Test](#_ke8cvtumjlqm) 5

[Bonus One: Application of Physiological Signals](#_5ujk0btptcjv) 5

[Bonus Two: Formal Logical Test of System Properties](#_i3dy01nr1x9o) 5

[**RESULTS**](#_7q4q4ak6x5ml) **7**

[Characterization of System One](#_qpqyzuoojmmf) 7

[Causality](#_cc4md8hcisw6) 7

[Memory](#_t84q4bx8f0cv) 8

[Linearity](#_26zjsuh4n1mq) 9

[Time Invariance](#_7kkf4ex7keyh) 13

[Characterization of System Two](#_o5kmov1852gg) 14

[Causality](#_gfqi5fqmmiyc) 14

[Memory](#_5ziovul8xax1) 15

[Linearity](#_8hbk1vbp1283) 17

[Time Invariance](#_6gmzkb1ag76u) 17

[Characterization of System Three](#_b0hcvpewk4zl) 20

[Causality](#_jyf8aor5va48) 20

[Memory](#_hunxlfngj7qd) 20

[Linearity](#_nw4ytlr3rxzk) 21

[Time Invariance](#_w8m72jvmc3um) 25

[Bonus One: Application of Physiological Signals](#_1rp0i6vbgwf4) 28

[System 1 Linearity Testing Using ECG and VGRF Data](#_flsmesws4dp6) 28

[System 2 Linearity Testing Using ECG and VGRF Data](#_i7j5sswcl6l7) 31

[System 3 Linearity Testing Using ECF and VGRF Data](#_af11u7fpmncq) 33

[Bonus Two: Formal Logical Test of System Properties](#_29jv5zav16eg) 36

[Summary of Findings](#_qex4i2dysf2q) 36

[**CONCLUSION**](#_p80j6zkapl9x) **37**

[**REFERENCES**](#_e37weqeqbmki) **38**

# PURPOSE

Engineers often encounter *black-box* systems which are systems composed of unknown equations. However, a single-input-single-output (SISO) system can be reverse engineered through analysis of its four fundamental properties (causality, memory, linearity, and time invariance) to characterize and potentially recreate the system. This report outlines the testing of the fundamental system properties of three discrete-time *black-box* systems using MATLAB 2022a.

# METHODOLOGY

## Causality Test

A causal system is when a system’s output at a time t1 does not depend on values of the input x(t) for t < t1[1]. In other words, a causal system is a system that does not depend on future inputs to change the current outputs. A causal system cannot have nonzero outputs until a non-zero input is applied under the condition of zero initial conditions. Therefore, we can use this idea to test for causality. We use randomized numbers for input function and watch for any cases of output changes before the inputs.

**Table 1.** Test cases used to assess causality in all systems.

| **#** | **Input** | **Description** |
| --- | --- | --- |
| 1 | [0,1,2,3,2,1,0] | Involve increasing and decreasing numbers in order |
| 2 | [0,4,-1,-3,-5,-2,3,1,1,1,2] | Randomized numbers with positive and negative numbers |
| 3 | [0, 0, 0, 0, 0, 0, 2, 1, 3, 2, 2, 2, 2] | Randomized numbers with positive and negative numbers |

*Notes: (i) For input #1, the discrete sample range, n, was -3 to 3 (inclusive); (ii) for input #2, the discrete sample range, n, was -5 to 5 (inclusive); (iii) for input #3, the discrete sample range, n, was -6 to 6 (inclusive).*

## Memory Test

A memoryless system does not have any outputs that are dependent on any inputs or outputs in the past or in the future. We test for it by using one test case as a reference input function, and changes the remaining test cases slightly. We compare the outputs and see if a minor change in the input can affect the output function.

**Table 2.** Test cases used to assess memory in all systems.

| **#** | **Input** | **Description** |
| --- | --- | --- |
| 1 | [0,0,3,4,2,1,-2,-5,-4,-1,0] | Randomized values with positive, negative, and zero values |
| 2 | [0,0,3,-2,2,1,-2,-5,-4,-1,0] | Changed the value at n = -2 |
| 3 | [0,0,3,-2,2,1,-2,-5,-4,5,0] | Changed the value at n = -2 and the value at n = +4 |

*Note: The discrete sample range, n, was -5 to 5 (inclusive) for each case.*

## Linearity Test

A system is *linear* if it is both *homogenous* and *additive* [1].

1. A system is homogeneous when multiplying the input by a number will result in the output being multiplied by the same number. When written as a mathematical expression:
2. A system is additive when summing two input signals will create an output that is the sum of the individual outputs of each input signal. When written as a mathematical expression:

A system is considered non-linear if a single case violates either homogeneity or additivity.

To test for linearity using MATLAB 2022a, homogeneity and additivity were checked separately. Homogeneity was tested by multiplying various input functions and their responses by positive and negative values. These scaled functions were then graphed and visually compared. If the graphs satisfied the equation for homogeneity, they were considered homogeneous. Once a system was determined to be homogeneous, it was then tested for additivity. Additivity was tested for by summing the previously used input functions and their responses. The combined functions and responses were then graphed and visually compared. If the graphs satisfied the equation for additivity, they were considered additive. Systems that were both homogeneous and additive were considered linear.

**Table 3.** Test cases used to assess linearity in all systems.

| **#** | **Input** | **Description** |
| --- | --- | --- |
| 1 | [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0] | Impulse Function |
| 2 | [0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1] | Unit Step Function |
| 3 | [0 0 0 0 0 0 0 0 0 0 0 1 2 3 4 5 6 7 8 9 10] | Unit Ramp Function |
| 4 | [-1.0000 -0.7071 0.0000 0.7071 1.0000 0.7071 -0.0000 -0.7071 -1.0000 -0.7071 0 0.7071 1.0000] | Sinusoidal Function |

*Note: The discrete sample range, n, was -10 to 10 (inclusive) for each case.*

To test homogeneity, these functions were scaled by 41 and -12. These numbers were determined randomly. To test additivity, the impulse and unit step functions were summed and the unit ramp and sinusoidal function were summed to create combination functions.

## Time Invariance Test

A system is *time-invariant* if for any input whose output is , the response to the shifted input is equal to for any . Otherwise, a system is *time-varying* if it is not time-invariant [1].

To test for time invariance in a discrete-time system in MATLAB 2022a, shifts relative to were used. With a range of [-5,5] for , one input (*x*) was centred at . Using the same sequence of nonzero values in *x*, these values were shifted to the right and left at varying magnitudes, as illustrated in Table 4. For the inputs, 5 nonzero values, including 2 negatives, were used to represent a holistic test case.

**Table 4.** Test cases used to test for time invariance in all systems.

| **#** | **Input** | **Description** |
| --- | --- | --- |
| 1 | x = [0 0 0 -2 2 1 3 -3 0 0 0] |  |
| 2 | x\_R\_1 = [0 0 0 0 -2 2 1 3 -3 0 0] |  |
| 3 | x\_R\_2 = [0 0 0 0 0 -2 2 1 3 -3 0] |  |
| 4 | x\_R\_3 = [0 0 0 0 0 0 -2 2 1 3 -3] |  |
| 5 | x\_L\_1 = [0 0 -2 2 1 3 -3 0 0 0 0] |  |
| 6 | x\_L\_2 = [0 -2 2 1 3 -3 0 0 0 0 0] |  |
| 7 | x\_L\_3 = [-2 2 1 3 -3 0 0 0 0 0 0] |  |

*Notes: (i) The discrete sample range, n, was -5 to 5 (inclusive) for each case; (ii)* *for naming convention, “R” and “L” were used to indicate right or left shifts, respectively, followed by the amount of shift.*

## Bonus One: Application of Physiological Signals

Discrete time signals for direct fetal ECG data and vertical ground reaction force (VGRF) data for those with Parkinson’s disease were obtained online [2,3]. These two signals were then used to test the linearity of systems 1-3 using the previously described methods.

## Bonus Two: Formal Logical Test of System Properties

The bonus 2 causality test was carried out by generating an all zero function and a randomly generated number function. We merged the two functions with the zero function at the beginning. We proceed to feed the merged input function into the system and return an output function. We compared the output function with the merged input function to check if there are any nonzero outputs at where there were zero inputs. If there were any nonzero outputs where there were zero inputs, the system was classified as non-causal. This test was then repeated several times to further validate the test. A system was determined to be causal if it passed the causality test for every randomly generated input function.

The bonus 2 memoryless test was determined by generating and duplicating a randomly generated function. We adjusted one number from the duplicated input function. We put both input functions into the system and received two output functions. We compared the output functions and determined if there is exactly 1 difference to tell if the system is memoryless or with memory. If there was not exactly 1 difference, the system was classified as with memory. This test was then repeated several times to further validate the test. A system was determined to be memoryless if it passed the memoryless test for every randomly generated input function.

The bonus 2 linearity test was performed by generating 2 randomly generated functions and 2 randomly generated scaling factors. The system responses to the non-scaled input functions were then recorded. Then the inputs were scaled and summed according to the generated scaling factors (function1 \* scaling\_factor1 + function2 \* scaling\_factor2). This newly scaled summed function was then input to a system and compared with the scaled and summer responses (response1 \* scaling\_factor1 + response2 \* scaling\_factor2). This can be represented with the equation: y[a1\*x1[n]+a2\*x2[n]] = a1\*y[x1[n]] + a2\*y[x2[n]]. If this equation was ever violated, the system was classified as non-linear. This test was then repeated several times to further validate the test. A system was determined to be linear if it passed the linearity test for every randomly generated input function.

The bonus 2 time invariance test was performed by randomly generating an input function and a time shift. Since the system only returns a sequence of amplitudes, the output should be the same whether the output is time shifted or not. In other words: y[x[n]] = system(n,x) and y[x[n] + a] = system(n,x). Using this property, the output of a randomly time shifted input was compared with the time shifted response. This test can be represented with the equation: y[x[n] + a] = y[x[n+a]]. If this equation was ever violated, the system was classified as time-varying. This test was then repeated several times to further validate the test. A system was determined to be time invariant if it passed the time invariance test for every randomly generated input function.

# RESULTS

## Characterization of System One

### Causality

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**Figure 1.** The input function [0,1,2,3,2,1,0] (left) and its respective output function after inputting into System 1.

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**Figure 2.** The input function [0,4,-1,-3,-5,-2,3,1,1,1,2] (left) and its respective output function after inputting into System 1.

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**Figure 3.** The input function [0, 0, 0, 0, 0, 0, 2, 1, 3, 2, 2, 2, 2] (left) and its respective output function after inputting into System 1.

Figure 1 shows that the inputs and outputs change at the same time. This is an early indicator that System 1 is a casual function, but more test cases are needed to confirm. Figure 2 supports the trend noted in Figure 1, where the overall graph did not change before the input changes. Despite some numbers having opposite signs for the input and output, this does not affect the causality criteria. Figure 3 further supports the trend of Figure 1 and Figure 2. From these three test cases above, we can see the similarity of the input and output functions changing at the same time. Therefore, System 1 is indeed casual.

### *Memory*

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**Figure 4.** The input function [0,0,3,4,2,1,-2,-5,-4,-1,0] (left) and its respective output function after inputting into System 1.

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**Figure 5.** The input function [0,0,3,-2,2,1,-2,-5,-4,-1,0] (left) and its respective output function after inputting into System 1.

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**Figure 6.** The input function [0,0,3,-2,2,1,-2,-5,-4,5,0] (left) and its respective output function after inputting into System 1.

Figure 4 depicts the first test case, which is used as a reference. We changed upcoming test cases to see what would happen if there are minor changes to this input reference.Figure 5 shows that changing only the input value at n = -2 causes only the output value at n = -2 to change. Figure 6 shows that changing only the input value at n = 4 (while keeping the change at n = -2 as described in Figure 5), causes only the output value at n = 4 to change. This is the only difference in outputs of test cases 2 and 3. We concluded that System 1 is not dependent on other inputs, thus it is memoryless.

### *Linearity*

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**Figure 7.** Plots comparing System 1’s actual response to the scaled impulse function with its expected response assuming System 1 is homogeneous.

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**Figure 8.** Plots comparing System 1’s actual response to the scaled sinusoidal function with its expected response assuming System 1 is homogeneous.

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**Figure 9.** Plots comparing System 1’s actual response to the combination function of the impulse and unit step functions with the expected response assuming System 1 is additive.

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**Figure 10.** Plots comparing System 1’s actual response to the combination function of the impulse and unit step functions with the expected response assuming System 1 is additive.

Figures 7–10 illustrate that System 1 passed every homogeneity and additivity test. Through visual confirmation, its actual responses and expected responses were always equal. Therefore System 1 is linear.

### Time Invariance

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**Figure 11.** Plots of input x[n] (left) with corresponding output y[x[n]] (right) at range -5 to 5 for System 1. Note that vertical axis scales are different.

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**Figure 12.** Plots of input x[n-1] (left) with corresponding output y[x[n-1]] (right) at range -5 to 5 for System 1. Note that vertical axis scales are different.

Figure 11 shows the plots of input x[n] with y[n], while Figure 12 shows the plots of input x[n-1] with y[n-1]. Despite a time shift of 1, the output y[n-1] is not a shifted plot of y[n]. This discrepancy demonstrates that System 1 is not time-invariant.

## Characterization of System Two

### Causality

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**Figure 13.** The input function [0,1,2,3,2,1,0] (left) and its respective output function after inputting into System 2.

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**Figure 14.** The input function [0,4,-1,-3,-5,-2,3,1,1,1,2] (left) and its respective output function after inputting into System 2.

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**Figure 15.** The input function [0, 0, 0, 0, 0, 0, 2, 1, 3, 2, 2, 2, 2] (left) and its respective output function after inputting into System 2.

Figure 13 shows a similar trend to that of System 1: the inputs and outputs change at the same time. Despite being a good indicator of causality, more test cases were implemented for verification. Figure 14 shows that, despite some inputs and outputs having different signs, the output did not become nonzero when input was zero. Figure 15 confirms that System 2 is a causal system because the system does not give nonzero outputs at zero inputs.

### *Memory*

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**Figure 16.** The input function [0,0,3,4,2,1,-2,-5,-4,-1,0] (left) and its respective output function after inputting into System 2.

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**Figure 17.** The input function [0,0,3,-2,2,1,-2,-5,-4,-1,0] (left) and its respective output function after inputting into System 2.

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**Figure 18.** The input function [0,0,3,-2,2,1,-2,-5,-4,5,0] (left) and its respective output function after inputting into System 2.

Figure 16 shows the first test case being used for reference, which is compared with the outputs of the other test cases. By previously described methods, Figure 17 shows that an input value change at n = -2 resulted in an output value change at only n = -2. Similarly, Figure 18 shows that, when compared with Figure 17, only changing the input value at n = 4 resulted in an output value change at n = 4. Therefore, through the three test cases, we confirmed that System 2 is a memoryless system.

### *Linearity*

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**Figure 19.** Plots comparing System 2’s actual response to the negatively scaled impulse function with its expected response assuming System 2 is homogeneous.

Figure 19 illustrates that System 2 failed the homogeneity test to the negatively scaled impulse function. Through visual confirmation, the actual and expected responses are not equal. Therefore System 2 is non-linear.

### Time Invariance

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**Figure 20.** Plots of input x[n] (left) with corresponding output y[x[n]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

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**Figure 21.** Plots of input x[n-1] (left) with corresponding output y[x[n-1]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

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**Figure 22.** Plots of input x[n-2] (left) with corresponding output y[x[n-2]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

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**Figure 23.** Plots of input x[n-3] (left) with corresponding output y[x[n-3]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

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**Figure 24.** Plots of input x[n+1] (left) with corresponding output y[x[n+1]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

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**Figure 25.** Plots of input x[n+2] (left) with corresponding output y[x[n+2]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

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**Figure 26.** Plots of input x[n+3] (left) with corresponding output y[x[n+3]] (right) at range -5 to 5 for System 2. Note that vertical axis scales are different.

Figures 20–26 show that time shifts of ±3 in inputs result in time shifts of ±3 in corresponding outputs. This trend aligns with the definition of time invariance. Therefore, System 2 was observed to be time-invariant.

## Characterization of System Three

### Causality

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**Figure 27.** The input function [0,1,2,3,2,1,0] (left) and its respective output function after inputting into System 3.

Figure 27 shows a violation of the causality principle: at n = 0, the system’s response had non-zero values before the input did. Therefore, System 3 is observed to be non-causal.

### Memory

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**Figure 28.** The input function [0,0,3,4,2,1,-2,-5,-4,-1,0] (left) and its respective output function after inputting into System 3.

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**Figure 29.** The input function [0,0,3,-2,2,1,-2,-5,-4,-1,0] (left) and its respective output function after inputting into System 3.

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**Figure 30.** The input function [0,0,3,-2,2,1,-2,-5,-4,5,0] (left) and its respective output function after inputting into System 3.

In Figure 28, the first test case was used for reference to compare with the other cases to determine if the system is memoryless or with memory. In Figure 29, the second test case (different from reference only at n = -2) had an output different from the first test case, especially with the surrounding output of the impacted input. Therefore, there was suspicion that this system has memory. In Figure 30, although the only value different from the second test case is at n = 4, the third test case has different output, such as at n = 5. Therefore, we have confirmed that System 3 has memory.

### *Linearity*

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**Figure 31.** Plots comparing System 3’s actual response to the scaled unit step function with its expected response assuming System 3 is homogeneous.

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**Figure 32.** Plots comparing System 3’s actual response to the scaled unit ramp function with its expected response assuming System 3 is homogeneous.

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**Figure 33.** Plots comparing System 3’s actual response to the combination function of the impulse and unit step functions with the expected response assuming System 3 is additive.

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**Figure 34.** Plots comparing System 3’s actual response to the combination function of the impulse and unit step functions with the expected response assuming System 3 is additive.

Shown in Figures 31–34, System 3 passed every homogeneity and additivity test. Through visual confirmation, its actual responses and expected responses were always equal. Therefore, System 3 is linear.

### Time Invariance

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**Figure 35.** Plots of input x[n] (left) with corresponding output y[x[n]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

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**Figure 36.** Plots of input x[n-1] (left) with corresponding output y[x[n-1]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

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**Figure 37.** Plots of input x[n-2] (left) with corresponding output y[x[n-2]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

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**Figure 38.** Plots of input x[n-3] (left) with corresponding output y[x[n-3]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

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**Figure 39.** Plots of input x[n+1] (left) with corresponding output y[x[n+1]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

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**Figure 40.** Plots of input x[n+2] (left) with corresponding output y[x[n+2]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

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**Figure 41.** Plots of input x[n+3] (left) with corresponding output y[x[n+3]] (right) at range -5 to 5 for System 3. Note that vertical axis scales are different.

Figures 35–41 show that time shifts of ±3 in inputs result in time shifts of ±3 in corresponding outputs. Like in System 2, this trend aligns with the definition of time invariance. Therefore, System 3 was observed to be time-invariant.

## Bonus One: Application of Physiological Signals

### System 1 Linearity Testing Using ECG and VGRF Data

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**Figure 42.** Plots comparing System 1’s actual response to the scaled ECG Data with its expected response assuming System 1 is homogeneous.

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**Figure 43.** Plots comparing System 1’s actual response to the scaled VGRF Data with its expected response assuming System 1 is homogeneous.

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**Figure 44.** Plots comparing System 1’s actual response to the combination function of the ECG and VGRF Data with the expected response assuming System 1 is additive.

Shown with Figures 42–44, System 1 continues to pass the homogeneity and additivity test while using ECG and VGRF Data as input. Through visual confirmation, its actual responses and expected responses were always equal. This further supports our conclusion that System 1 is linear.

### System 2 Linearity Testing Using ECG and VGRF Data

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**Figure 45.** Plots comparing System 2’s actual response to the negatively scaled ECG Data with its expected response assuming System 2 is homogeneous.

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**Figure 46.** Plots comparing System 2’s actual response to the negatively scaled VGRF Data with its expected response assuming System 2 is homogeneous.

Shown by Figure 45 and Figure 46, System 2 failed the homogeneity test to the negatively scaled ECG and VGRF Data, respectively. Through visual confirmation, the actual and expected responses are not equal. This further supports our conclusion that System 2 is non-linear.

### System 3 Linearity Testing Using ECF and VGRF Data

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**Figure 47.** Plots comparing System 3’s actual response to the scaled ECG Data with its expected response assuming System 3 is homogeneous.

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**Figure 48.** Plots comparing System 3’s actual response to the scaled VGRF Data with its expected response assuming System 3 is homogeneous.

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**Figure 49.** Plots comparing System 3’s actual response to the combination function of the ECG and VGRF Data with the expected response assuming System 3 is additive.

Shown in Figures 47–49, System 3 continues to pass the homogeneity and additivity test while using ECG and VGRF Data as input. Through visual confirmation, its actual responses and expected responses were always equal. This further supports our conclusion that System 3 is linear.

## Bonus Two: Formal Logical Test of System Properties

The results of our formal logical tests on System0 revealed that it is causal, with memory, linear, and time invariant. By examining the matlab file, we determined that the system is:

y[n] = 0.25\*x[n] + 0.5\*x[n-1] + 0.25\*x[n-2]. Through analysis of the equation, we can confirm that the results of our formal logical test are correct for system0.

The results of our formal logical tests for systems 1-3 each agree with and support our conclusions on the properties of systems 1-3.

## **Summary of Findings**

Table 5 below summarizes our findings after conducting tests for causality, memory, linearity, and time invariance on the three systems.

**Table 5.** Properties of the three systems based on MATLAB-coded tests.

|  | **System One** | **System Two** | **System Three** |
| --- | --- | --- | --- |
| **Causality** | Causal | Causal | Non-causal |
| **Memory** | Memoryless | Memoryless | Has memory |
| **Linearity** | Linear | Non-linear | Linear |
| **Time Invariance** | Time-varying | Time-invariant | Time-invariant |

# CONCLUSION

We successfully ran tests for causality, memory, linearity, and time invariance on three systems using MATLAB 2022a. Additionally, we applied our understanding of these properties by testing discrete-time signals from direct fetal ECG data and vertical ground reaction force (VGRF) data for those with Parkinson’s disease. We finally conducted a formal logical test for each 4 properties, while taking into account the finite precision mathematics of MATLAB 2020a.

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